What initial velocity model do we need for full waveform inversion?
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SUMMARY

In the context of velocity model building, we examine if the velocity models obtained after first-arrival traveltime tomography are accurate enough for subsequent full waveform inversion of reflected energy. For that purpose, we test the quality of the velocity model obtained by first-arrival traveltime tomography on the BP salt dome model. Several 1-D inversions are conducted in two different zones. In the simplest zone corresponding to smooth velocity models, the tomographic models are good enough for waveform inversion with realistic frequency contents. In the complex part going through a salt body, one need very low frequencies (starting at around 1 Hz) or a further refinement of the tomographic model.
Introduction
For successful applications of full waveform inversion, it is critical to generate a sufficiently accurate starting model in order to allow the misfit function to converge to the global minimum. A “sufficiently accurate” initial model means that the synthetic waveforms must match the observed data to within less than a half-cycle; otherwise a local minimum of the misfit function may be encountered.
A common factor of a number of leading research groups working on waveform inversion is the combination of first-arrival traveltime tomography and waveform inversion applied on large-offset data (e.g. Pratt and Brenders, 2004, Operto et al., 2004). These algorithms can be seen as a way to unravel the complexities of refracted arrivals in order to provide constrained velocity models.
In the same framework, we examine if the velocity models obtained by first-arrival traveltime tomography in the context of wide-angle acquisition are accurate enough for subsequent full waveform inversion of reflected waves. We assess the accuracy of the tomographic model in relation with the minimum frequency content of the seismic data for a successful waveform inversion by performing several simple 1-D benchmark tests on the BP salt dome synthetic model.

Methodology
First-step: first-arrival traveltime tomography
We use a recently suggested refraction traveltime tomography algorithm formulated as a non-linear optimization problem and based on the adjoint state method to compute the gradient of the misfit function (Taillardier et al., 2007). This inversion method was designed to overcome computational limitations that could affect classical refraction traveltime tomography algorithms in case of dense acquisition and/or large velocity models. The computational benefits using the adjoint state method, compared to classical algorithms, are a lower memory requirement, a straightforward and efficient parallelization and an effortless implementation. These properties have already been assessed and validated for 2-D acquisitions. For details on the algorithm, we refer to Sei & Symes 1994, Leung & Qian 2006 and Taillardier et al., 2007.

Second step: full waveform inversion
In the approach conducted here, we use the time domain to generate the finite difference synthetic shots in a particular velocity model (Noble, 1992), with an order 8 in space and 4 in time (variable density acoustic wave equation). To compute the gradient of the misfit cost function, we switch to the frequency domain and use the Born approximation (Miller et al., 1987)

\[ \delta P(s, r, \omega) = -\omega^2 \int dx \, P_0(s, x, \omega) \cdot \delta m(x) \cdot G_0(x, r, \omega), \]

where \( P_0 \) is the reference shot, \( G_0 \) the Green’s function computed in the current model, \( \delta m(x) \) a particular velocity perturbation \((\delta n=\Delta \xi / c^2)\) and \( \delta P \) the perturbation to be added to the current shot to match the observed shot. The variables \((s, r, x, \omega)\) respectively correspond to the source and receiver positions, the position within the model and the frequency. Equation (1) can be written as \( b = Ay \). As we only deal with 1-D models, it is possible to get \( y \) with \( y = (A' A + \epsilon I)^{-1} A' b \), where \( \epsilon \) is a small damping number. Once the velocity perturbation is obtained, we regenerate a new shot with the time-domain finite difference code. By going to the frequency domain and solving equation (1), we avoid to define a preconditioning for the misfit cost function. This preconditioning is essential to speed-up the convergence and exploit subtle changes in the residuals during the minimization process.

Application
The BP 2-D synthetic dataset is used as a reference model for our benchmarks (Billette and Brandsberg-Dahl, 2005). We first perform a 2-D traveltime tomography on first arrivals. We then extract two profiles for the waveform inversion tests. First profile corresponds to an area
between the two salt bodies with smooth velocity variations. Second profile goes through a salt body with a large velocity jump from 2.5 to 4.8 km/s (Figure 1).

**Traveltime tomography**

For the details on the tomographic inversion, we refer to Taillardat et al., 2007. The starting model is a simple constant velocity gradient (Figure 1, top). The algorithm combined with a conjugate gradient optimization technique converges to a final velocity model displayed in Figure 1, middle. Ray paths computed in the true velocity model (Figure 1, bottom) allow defining an area (in grey colour) where first arrival traveltimes contain no information about the model. The inverted model is smooth by nature and fits the main trends of the true model (Figure 2).

![Fig. 1: Initial (top), inverted (middle) and exact (bottom) velocity models after and before the tomographic inversion. The dark area indicates the zone where the rays do not penetrate.](image1)

![Fig. 2: Velocity profiles for x=25 km and x=37.5 km, for the initial (red – constant gradient), inverted (green – smooth model) and exact (black – with hard interfaces) model](image2)

**Full waveform inversion**

In order to assess the quality of the tomography result, we use the two velocity profiles from Figure 2. A dense reflectivity was added to the exact profile to mimic the original shot gathers. Finite difference shots were generated with a time-domain modelling code, without a free surface. The final tomographic model is used as the initial model for the full waveform inversion.

In the sediment part with no strong velocity contrast, the maximum offset used is 4.5 km. In spite of the inaccuracy of the initial model, the full waveform inversion retrieves the exact profile (Figure 3). For this case, we were able to converge with realistic frequencies above 3.0 Hz. That minimum frequency value is possibly even higher.

In the presence of salt with high velocity contrasts (a factor about 2 at the top salt interface), full waveform inversion is able to converge to the correct model only if the data contain frequencies starting at 1.1 Hz (Figure 4, left). The maximum offset used is 6 km. Different
snapshots in the initial, final and exact models show that the first arrival, but also reflected energy, in particular on the top salt, are well retrieved after inversion (Figure 5). With a minimum frequency at 2.0 Hz, the inversion leads to a local minimum (Figure 4, right). These results are consistent with the one derived from the cost functions displayed in Figure 6. The misfit function is computed for a series of velocity model linearly interpolated between the initial and exact models. In the presence of salt and for a minimum frequency of 2.0 Hz, a local minimum is observed. That is not the case for minimum frequencies lower than 1.1 Hz (Figure 6, right).

**Fig. 3:** Initial (smooth profile), inverted (solid line) and exact (dashed line) velocity profiles after a 1-D inversion (position x=25 km) starting from the model provided by the tomography process. The minimum frequency in the input data is 3.0 Hz.

**Fig. 4:** Initial (dotted line), inverted (solid line) and exact (dashed line) velocity profiles after a 1-D inversion (position x=37.5 km) starting from the model provided by the tomography process. The minimum frequency in the input data is 1.1 Hz (left) and 2.0 Hz (right).

**Fig. 5:** Snapshot for t=2.6 s, computed in the initial (left), inverted (middle) and exact (right) velocity models. The reflection on the top salt is nicely explained after inversion.
Fig. 6: Cost functions measured for different velocity models. A model is linearly interpolated between the initial and exact velocity model. For the velocity profile through the sediment part, the minimum frequency used is 3 Hz (left). For the profile though the salt, the minimum frequencies are 0.5, 1.1 and 2.0 Hz (right, dotted, solid and dashed lines).

Discussion and conclusions
A velocity model has been obtained by first-arrival traveltime tomography and used as an initial model for subsequent full waveform inversion. For small velocity contrasts, the full waveform inversion on reflected energy with limited offsets correctly converges to the global minimum for a minimum frequency of 3 Hz or possibly even higher. For large velocity contrasts, low frequencies starting at 1 Hz in the input data are required to recover the velocity jump. In order to start with higher frequencies, one would be need to further refine the velocity model obtained by first-arrival traveltime tomography. This is in principle possible by analyzing reflected energy above top salt with a standard ray-based tomography process. This approach is consistent with later full waveform inversion that – in this article – mainly deals with reflected energy.

One could question the validity of such an approach in case of anisotropic Earth model. Refracted energy indeed mainly propagates horizontally whereas reflected energy is sensitive to vertical velocities.

References
Leung, S. and Qian, J., 2006, An adjoint state method for three-dimensional transmission traveltime tomography using first arrivals, Communications in Mathematical Sciences, 4, 249–266.